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Brief Report

The Effects of Preservice Elementary School Teachers' Accurate Self-Assessments in the Context of Whole Number

Eva Thanheiser
Portland State University

Elementary prospective school teachers (PSTs) often struggle to understand why they need to relearn the mathematics that they think they already know. In this set of replication studies, I address this struggle in 3 ways: First, by increasing and varying the participant pool, I replicate Thanheiser's (2009) study, which shows that PSTs do not yet understand number in a way that enables them to teach it. Second, I introduce, validate, and examine the effect of using a survey instead of interviews, thus changing the methods of the original study. Third, I move beyond Thanheiser's original study and show that an interview designed to help PSTs assess their own knowledge accurately correlates with more sophisticated conceptions at the end of the course. Based on these findings, I posit that such an interview could be used to help PSTs learn the mathematics that they need to teach.

Keywords: Assessment; Content knowledge; Counting numbers; Metacognition; Natural numbers; Preservice teachers; Whole numbers

"Number is simple; we should concentrate on the more difficult topics."
—*Mathematics teacher educator*

"But I already know all the math I need to know to teach elementary school."
—*Preservice teacher*

Thanheiser's (2009) Study and the Need for Replication

Although prospective school teachers (PSTs) can execute algorithms, they struggle to explain the mathematics underlying them (Ball, 1988b; Ma, 1999; Thanheiser, 2009, 2010). This disconnect between procedures and concepts may lead to a procedural view of mathematics as a disjointed set of rules that one must memorize rather than a conceptual view of mathematics as an integrated system that makes sense (Hiebert & Lefevre, 1986). A procedural view does not support a student's *productive disposition*, the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (Kilpatrick, Swafford, & Findell, 2001, p. 116). If PSTs view mathematics as a set of disconnected procedures, they may perpetuate that view in their future classrooms (Battista, 1994) because teachers who cannot make sense of mathematics themselves cannot teach it in a sense-making fashion and, thus, may focus on procedures. Mathematics teacher educators (MTEs) must, therefore, recognize that many PSTs do not yet adequately understand number at the beginning of their content courses and must integrate a focus on number into their courses.

I introduced a framework for PSTs' conceptions of number (Thanheiser, 2009) to address the need for MTEs to understand these conceptions. In this framework, I categorized PSTs' conceptions for multidigit whole numbers into four distinct conceptions (two correct and two incorrect). PSTs with a *reference-units conception* are able to flexibly see each digit with respect to the unit type it represents (ones, tens, hundreds, etc.) and are able to relate those units to one another. Thus, PSTs with a reference-units conception see the 5 in 527 as 5 hundreds, 50 tens, or 500 ones. With this conception also comes the flexibility to regroup, for example, 1 hundred into 10 tens when subtracting. PSTs with a *groups-of-ones conception* see all digits with respect to the groups of ones that they represent—that is, they see the 5 in 527 as 500 but not as 50 tens. The groups-of-ones conception is a subconception of the reference-units conception. PSTs with a *concatenated-digits-only conception* see a number as several single digits arranged in succession. So, 527 is a 5 next to a 2 next to a 7, and together they make 527. Single digits form larger numbers similar to the way letters form words (e.g., cat is composed of a *c* next to an *a* next to a *t*). PSTs with a *concatenated-digits-plus conception* draw on a mix of a concatenated-digits-only conception and one of the other conceptions. See Figure 1 for one interpretation of 527 and its regrouping in the context of subtraction within the various conceptions.

In Thanheiser (2009), I used the framework above to (a) establish that two thirds of the PSTs in that study held incorrect conceptions and (b) explain why PSTs

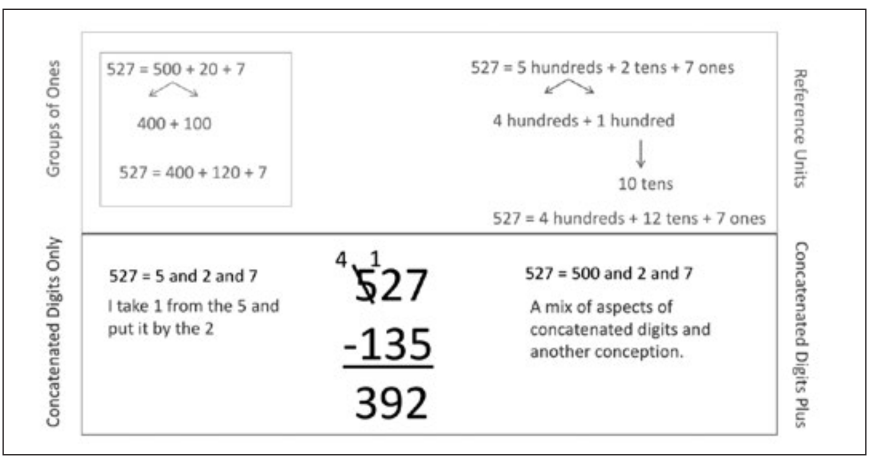


Figure 1. Examples of Thanheiser's (2009) four conceptions of multidigit whole numbers, two correct (upper two) and two incorrect (lower two).

struggle to explain the algorithms on the bases of the conceptions that they hold. The two concatenated-digits conceptions do not allow for explaining the place-value ideas underlying the algorithms (for a more detailed discussion, see Thanheiser, 2009, 2012). The framework was based on interviews with 15 PSTs, all from the same participant pool: students entering their first content course for PSTs at the same institution. A limitation of the original study was that "the data pool . . . [was] too small to indicate general trends" (Thanheiser, 2009, p. 277). I addressed this limitation with the replication study.

Replication studies are essential for generalizing from a small population to a larger population (Schmidt, 2009) and making statements beyond the first context (Schoenfeld, 2007). Replication studies are essential in a field in which small sample sizes dominate (Melhuish, 2018); however, only about 1 in 1,000 papers in the major educational journals is a replication study (Makel & Plucker, 2014).

As part of the replication, I also examined feasibility. Because a one-on-one interview with each student to determine his or her conceptions is time consuming for both the MTE and the PSTs, I examined whether a survey could replace the interview to assess PSTs' conceptions.

After becoming aware of the PSTs' conceptions, I began to wonder whether the PSTs were aware of their own incoming conceptions and how this awareness would influence their engagement with the course. This question led me to examine whether the PSTs think that their content knowledge (before the course in which they are enrolled) is adequate to teach K–3 and whether an interview or a survey could help the PSTs accurately assess their own understanding. I discuss the need for both assessment and self-assessment in the next section.

The Need for Assessment and Self-Assessment

Accurate assessments of student conceptions are essential for both teachers and students in a classroom. Teachers can use assessments to develop their lessons around student conceptions (Bransford, Brown, & Cocking, 1999; Conference Board of the Mathematical Sciences, 2001). Students can use assessments to recognize what they do and do not know and, thus, develop mathematical integrity (DeBellis & Goldin, 1999; Goldin, 2002) and metacognition. Metacognition, which includes the "ability to monitor one's own understanding and problem-solving activity" (Kilpatrick et al., 2001, p. 118), has been shown to be a powerful predictor of learning (Veenman, Van Hout-Wolters, & Afflerbach, 2006; Wang, Haertel, & Walberg, 1990).

Both MTEs and PSTs may inaccurately assess the PSTs' incoming knowledge. Elementary school mathematics is often viewed as simple or easy (Ambrose, 2004; Ball, 1988a; Wu, 2009); therefore, adults (including PSTs) are expected to know and be able to explain it. Because PSTs are able to perform algorithms (Ball, 1988b; Ma, 1999; Thanheiser, 2009, 2010), they are often assumed to possess the underlying knowledge needed to explain them; thus, MTEs may spend little time on examining whole numbers in their courses.

PSTs themselves often equate remembering and applying procedures with understanding mathematics (Ball, 1990; Graeber, 1999; McDiarmid, 1990; Smith, 1996). If PSTs think that the mathematics they know is sufficient for them to teach, they may see no need to relearn it (Philipp et al., 2007). Furthermore, they believe that if they have more to learn, they will learn it after they are in their K–5 classrooms (McDiarmid, 1990). PSTs with such attitudes may see no need to engage in the college mathematics classroom.

For PSTs to recognize the need to learn more mathematics, they must become aware (Mason, 1998) of their lack of the mathematical knowledge needed to teach in elementary school (Adler & Ball, 2008); otherwise, they are unlikely to engage in class activities (Pintrich, 2002):

> If students do not realize they do not know some aspect of factual, conceptual, or procedural knowledge, it is unlikely they will make any effort to acquire or construct new knowledge. Accordingly, we stress the need for teachers to help students make accurate assessments of their self-knowledge, not inflate their self-esteem. (Pintrich, 2002, p. 222)

Thus, it is essential for both MTEs and PSTs to recognize that the PSTs lack the mathematical knowledge they need to teach elementary school so that MTEs provide opportunities for the PSTs to (re)learn the mathematics and so that PSTs are motivated to engage in the activities of their content courses.

In prior work, Thanheiser, Philipp, Fasteen, Strand, and Mills (2013) used an interview with PSTs not only to inform the MTE of the PSTs' conceptions but also to help the PSTs realize that

> (a) there is something to learn beyond procedures, (b) their own knowledge is limited and they need to know more to be able to teach, and (c) engaging in the mathematical activities in their content courses will lead them to learning important content. (p. 137)

We posited that an interview through which the PSTs recognize their knowledge limitations can set them on a trajectory to develop more sophisticated conceptions during their content course by motivating them to engage with the content. In this Brief Report, I extend this work by examining the effect of such an interview on PSTs' conceptions and comparing the use of an interview with the use of a survey to assess conceptions at the beginning of the course.

In this article, I report on a set of studies designed to answer the following research questions:

1. Do the findings on the categorization of PSTs' conceptions of multidigit numbers (Thanheiser, 2009) generalize beyond the original context?
2. How can a survey instead of an interview be used to assess PSTs' conceptions?
3. Do PSTs think that their knowledge of mathematics when they enter their content and methods courses is sufficient for teaching elementary school?
4. Does an interview designed to help the PSTs assess their own understanding affect their learning?

The Set of Studies

In this Brief Report, I discuss data analyzed across a set of five studies (Thanheiser, 2009, and four additional replication studies) designed to answer the four research questions stated above. In these studies, I examined conceptions of multidigit whole numbers that PSTs held before and after their first content or methods course. As such, the set of four additional studies may serve to replicate the findings of Thanheiser (2009) in terms of the categorization of PSTs' conceptions by scaling up (larger participant pool) and out (at different universities and at different time points in PSTs' education). The replications in this set of studies are located between operational and constructive replications (Lykken, 1968). In *operational replications*, "one strives to duplicate exactly just the sampling and experimental procedures given in the first author's report of his research" (Lykken, 1968, p. 155), whereas in *constructive replications*, "one deliberately avoids imitation of the first author's methods" (Lykken, 1968, pp. 155–156). The methods in these four replication studies (Studies 2–5) had both similarities to and differences from those used in Thanheiser's (2009) study (Study 1).

See Table 1 for an overview of the school, U.S. location, assessment time point in the PSTs' education, method used to assess the conceptions (interview or survey), and number of participants across the five studies.

Methods Across Studies

Across all five studies, I was the MTE (teacher of record), and I conducted the vast majority of the interviews.[1] All interviews and surveys were double coded by

Table 1
Context Across Studies 1–5

	Study 1	Study 2	Study 3a	Study 3b	Study 4	Study 5	Total
School	School 1	School 2	School 2	School 2	School 3	School 3	
U.S. location	SW	NE	NE	NE	NW	NW	
Assessment pre-	Content	Methods	Methods	Methods	Content	Content	
Method	Interview	Survey	Survey	Interview	Interview	Survey	
Number of participants	15	33	25[a]	26	74	23	171

[a]Studies 3a and 3b are based on the same participants, but one participant did not complete the survey, creating a discrepancy of one. For the overall replication study, Study 3b is used; thus, Study 3a is not counted in the total column.

[1] In some studies, I had a teaching or research assistant work with me. In those cases, the assistant would conduct a few interviews after extensive training, which included viewing recorded interviews, observing during interviews I conducted, and reviewing their interviews with me after they had conducted them.

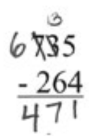
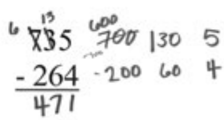
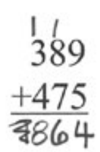
myself and research assistants, and interrater reliabilities for all studies are reported in Tables 3 and 6; all disagreements were resolved through discussion.

I used the same tools to measure PSTs' conceptions at the beginning and at the end of the course. Again, all data were double coded and interrater reliabilities are reported below in Tables 3 and 6.

Methods for Research Question 1

To answer the first research question—Do the findings on the categorization of PSTs' conceptions of multidigit numbers (Thanheiser, 2009) generalize beyond the original context?—I assessed a total of 171 PSTs across Studies 1–5 (15 in the original study, Study 1; 156 across the replication studies, Studies 2–5). The PSTs all attended large public universities. In contrast to the interviews in Study 1 (which were exploratory in nature and included two 60–90-minute interviews with each PST to develop a framework), the interviews in the replication studies were 10–15 minutes long and were focused on essential questions for identifying conceptions (see the protocol for this interview in Appendix A). All interviews were coded independently by two coders who watched the interviews or read transcripts and identified on which of the four conceptions (reference units, groups of ones, concatenated digits plus, or concatenated digits only) the PSTs were drawing to answer the questions. Example transcripts and coding are shown in Table 2 (for more detailed information, see Thanheiser, 2009).

Table 2
Coding Examples

Context	Transcript sample	Coding
	Instead of the 700, I made this 600, and then I added 10 tens so the hundred went here [to the tens] but it was really 10 tens plus the 3 that I already had, so 13 tens, which is really 130.	*Reference units*
	I know that this is actually, when I cross out—when I cross it out—when I write it like this: 700, 30, 5 minus 200, 60, and 4; since I can't do this [30 minus 60], I'm taking out 100 here [from the 700] . . . and making it 600 . . . and I'm actually adding 100 here [to the 30] to make it 130 so I can subtract it.	*Groups of ones*
	9 plus 5 is 14. Yes, 14. So, put the 4 here and carry the 1 on top and—8 plus 7 would be 15, and then add the 1 here [above the 3], since we carried it from here, so 16. And, again, carry the 1 here. And 4 plus 3's seven. Plus 1 would be 8. And I guess we carry the 1 over because we can't put a 14 here 'cause—we wouldn't—we have too big of a number.	*Concatenated digits only*

The following variations in methods occurred across the studies. First, the geographical region of the participant pool varied: Study 1 was conducted in the Southwest, Studies 2 and 3 were conducted in the Northeast, and Studies 4 and 5 were conducted in the Northwest of the United States. The goal for this variation was to be able to make statements beyond the small number and single geographical location of Study 1.

Second, the timing within the PSTs' educations varied. In Studies 1, 4, and 5, the PSTs were assessed immediately before their first content course. PSTs at both of these study locations were required to complete at least three content courses before being admitted into the graduate teacher education program. In Studies 2 and 3, PSTs were assessed before entering their methods courses, which are typically taken toward the end of the PSTs' education in the United States. Participants in Studies 2 and 3 were required to complete two semester-long mathematics classes before entering their methods courses. Although many universities now offer (and require) mathematics content courses specially designed for teachers, PSTs at this school were not required to take such specially designed courses. My goal for examining PSTs at the beginning of the methods course was to broaden the scope of the study from content to methods courses. Interviews were scheduled on the first day of class and were typically conducted within 1 week.

Methods for Research Question 2

To answer the second research question—How can a survey instead of an interview be used to assess PSTs' conceptions?—I developed a survey designed to assess the same conceptions as the interview. The survey questions can be found in Appendix B, and more information about the survey can be found in Thanheiser (2010). My initial goal for developing the survey was to examine the feasibility of using it to replace the interview, which would ease the workload on both the MTE and the PSTs in assessing the PSTs' conceptions at the beginning of the course. Whereas the interview requires a one-on-one setting with each PST, all PSTs can complete the survey at the same time. The survey was piloted in Study 2 and validated in Study 3. PSTs in Study 3 participated in both the survey (Study 3a) and the interview (Study 3b) to assess the PSTs' conceptions, with 92% agreement between interpretations (correct vs. incorrect conception) drawn on the basis of the survey and those drawn on the basis of an individual interview. The survey was also used to examine the effect of each (interview and survey) on the PSTs' learning. This is addressed in further detail in Methods for Research Question 4. The PSTs completed the survey immediately preceding the place-value instruction in the methods section or on the first day of class in the content course.

Methods for Research Question 3

To answer the third research question—Do PSTs think that their knowledge of mathematics when they enter their content and methods courses is sufficient for teaching elementary school?—I asked PSTs in Studies 4 and 5 that question (see Figure 2 for the prompt that they were given). This question was posed on the first

Do you think that your current mathematical knowledge is sufficient for you to teach K–3?

(a) Yes (b) No

Please explain.

Figure 2. Question about whether PSTs know enough math to teach K–3.

day of class (before the individual interviews or as part of the survey) in a brief survey of background information completed by the students. The responses to this question allow for comparison between the PSTs' self-assessments of their mathematics understanding and my assessment of their conceptions.

Methods for Research Question 4

To answer the fourth research question—Does an interview designed to help the PSTs assess their own understanding affect their learning?—I introduced the interview in Study 4 to the PSTs as having two purposes: to help their teacher (me) learn about their thinking and as a tool for the PSTs themselves to gauge their knowledge at the beginning of the course. On the first day of class, PSTs were told that I would conduct an interview with each of them to help me plan the class and to help them better understand themselves. During the interview, I told them that I would ask questions and push them so that we would both understand what they do and do not yet know. If PSTs asked for a resolution of their unresolved points during or after the interview, I explained that these would be the topics of discussion during our class. The PSTs were also told that they would watch the videotape of their interview at the end of the course to reflect on their knowledge and learning.

Before I characterized the interviews as helping both the PST and myself to learn about the PST's understanding, the PSTs seemed to view the interview as a tool for the instructor and did not seem concerned about their responses. In the context of both of us learning about the PST's understanding, some PSTs commented after the interview that they looked forward to being able to explain the things that they could not explain in the interview (for more on this, see Thanheiser, 2017). The goal was to allow the PSTs to notice their own limitations when responding to the interview questions.

Studies 4 and 5 examined the difference between conducting an interview and giving a survey at the beginning of the course. In Study 4, I conducted an interview with each PST in the beginning. In Study 5, the PSTs completed an initial survey. Other variations were held constant across these studies (same geographical region, same timing, same curriculum, and same treatment except for interview vs. survey).

Results

Research Question 1: Generalizability Beyond the Original Context

In my previous work (Thanheiser, 2009), 5 of the 15 interviewed PSTs (33%) held a correct conception and 10 (67%) held an incorrect conception at the start of their first content course. I described these conceptions in detail and linked them to the PSTs' inabilities to explain why the standard algorithms (or other methods) for addition and subtraction yield correct answers by relating the PSTs' conceptions to the explanations that they were able to give. This work is represented as Study 1 in this set of studies (see Table 1). The results, that most PSTs enter their content courses with incorrect conceptions of multidigit whole numbers, were found across all four replication studies (Studies 2–5; see Table 3). In fact, for all of the studies, the percentage of PSTs who held a correct conception was far below the 33% found in the original study. In total, of the 171 PSTs participating across Studies 1–5, only 31 (18%) held correct conceptions, whereas 140 (82%) held one of the incorrect conceptions.

Together, Study 1 (the original study) and Studies 2–5 show that the categorization of the PSTs' conceptions in the original study (Thanheiser, 2009) is generalizable beyond its original context to different universities, at different time points in the PSTs' education, and in different geographical regions. The group of five studies (Studies 1–5) provides a broader understanding than the original study (Study 1) of the conceptions that PSTs hold when entering their mathematics content and methods classes. Although the four replication studies differed in terms of the percentages of PSTs who drew on correct conceptions at the beginning of their courses, ranging from 9% to 27%, no study showed that more than one third of the PSTs, the number found in the original study, held a correct conception. These results could inform task design for PST courses. Recognizing the conceptions that most PSTs hold when entering their mathematics content and methods courses is the first step in helping them develop more sophisticated conceptions.

Research Question 2: Survey versus Interview

An immediate concern after the original study (Thanheiser, 2009) was the time required for the MTE to interview each PST within a brief period at the beginning of the course (before the topic is discussed). Thus, the feasibility of a survey was considered. The survey was designed and piloted in Study 2 and validated in Studies 3a and 3b (see Table 3). Validation of the survey was done at the correct conception versus incorrect conception level; thus, the survey allows the MTE to categorize the PSTs' conceptions as correct (reference units or groups of ones) versus incorrect (concatenated digits only or concatenated digits plus). As such, the survey can provide the instructor of a course a quick overview of the class at the beginning of a course and a way to measure growth (from incorrect to correct conceptions). The survey has two limitations. First, it does not provide a categorization across all four conceptions. Second, it does not allow for follow-up questions that may enable PSTs to recognize the limitations of their own understanding.

Table 3
PSTs' Conceptions at the Beginning of the Course Across Studies 1–5 With Interrater Reliability

	Study 1	Study 2	Study 3a[a]	Study 3b	Study 4	Study 5	Total
Number of participants	15	33	25	26	74	23	171
Conception							
Reference units	3			5	5		
		3[b]	5			3	31 (18%)
Groups of ones	2			2	8		
Concatenated digits plus	7			13	34		
		30	20			20	140 (82%)
Concatenated digits only	3			6	27		
Percentage correct pre	33%	9%	20%	27%	17%	13%	18%
Interrater reliability	87%	86%	86%	84%	85%[c]	85%	

[a]Studies 3a and 3b are based on the same participants, but one participant did not complete the survey, creating a discrepancy of one. This participant was categorized as holding a concatenated-digits-only conception in the interview. For the overall replication study, Study 3b is used; thus, Study 3a is not counted in the total column.

[b]In Studies 2, 3a, and 5, the correct conceptions (reference units and groups of ones) are grouped together and the incorrect conceptions (concatenated digits plus and concatenated digits only) are grouped together.

[c]This interrater reliability for some of this data was calculated only to the level of correct versus incorrect rather than by conception.

Research Question 3: PSTs' Self-Perceptions

In Studies 4 and 5, I examined the PSTs' perceptions of their own mathematics preparation at the beginning of their content courses. All PSTs in these studies were asked whether they thought that they knew enough mathematics to teach Grades K–3.

Of the 97 PSTs who were asked this question, 65 (67%) answered affirmatively, stating that they thought that their knowledge of mathematics was already adequate (see Table 4). More specifically, of the 81 PSTs whose conceptions were categorized as incorrect, 52 (64%) thought that they knew enough math to teach. Table 4 shows the breakdown of responses with respect to the categorized conception for each of the PSTs in Studies 4 and 5.

This discrepancy between PSTs' perceived knowledge of elementary mathematics and their displayed conceptions in the place-value assessment is important to recognize because PSTs who think that their knowledge is sufficient for teaching may not engage meaningfully in the course activities. By recognizing the limitations of their own understanding of elementary mathematics, PSTs may realize the value of their opportunities to learn important mathematics in the course.

Table 4
PSTs' Perceptions of Mathematics Knowledge Related to Their Conceptions

	Correct conception	Incorrect conception	Total
Stated that they knew enough math to teach K–3	13	52	65 (67%)
Stated that they did not yet know enough math to teach K–3	3	29	32 (33%)
Total	16	81	97

Research Question 4: Effect of an Interview or Survey on Learning

To examine the effect of an interview versus a survey at the beginning of the content course, I compared Study 5 (in which the PSTs completed a survey and no interview) with Study 4 (in which the PSTs were interviewed). I focused on Studies 4 and 5 because the participants were the same population of PSTs at the same institution and because the interview was used as both an assessment for the instructor and an assessment intended specifically to help the PSTs recognize their own limitations. The PSTs were told before the interview that it was intended to record their current understanding and that they would review it at the end of the course to compare their understanding before and after the course. Immediately following the interview, the PSTs were asked to reflect on the interview (see sample reflection questions in Table 5). For a detailed description of this interview and reflection, see Thanheiser et al. (2013).

I compared the proportion of PSTs whose conceptions changed from incorrect at the beginning of the course to correct at the end of the course in Studies 4 and 5. In Study 5, the surveys showed that 20 PSTs initially held incorrect conceptions, and 11 of these PSTs held correct conceptions at the end of the course (see Table 6). In Study 4, the interviews showed that 61 PSTs initially held incorrect conceptions, and 54 of these held correct conceptions at the end of the course (see Table 6). I conducted a chi-square test of independence to test for the equality of proportions between the two samples of PSTs. The proportion of students with the interview treatment (Study 4) whose conceptions changed from incorrect to correct (54/61 = 0.89) was significantly greater than the proportion of students with survey treatment (Study 5) whose conceptions changed from incorrect to correct (11/20 = 0.55), $\chi^2(1, n = 81) = 11.000$, $p < .001$.

Although several differences between the groups may account for the difference in proportion of PSTs whose conceptions changed from incorrect to correct, I focus on one in this discussion: the presence (in Study 4) or absence (in Study 5) of an interview at the beginning of the course to assess the PSTs' conceptions. The interview highlighted to the PSTs themselves that they still had mathematics to learn. I share two typical PST quotes illuminating this sentiment.

> I think, without the interview, I might have gone into class a bit cockier (if possible) and possibly have taken the class less seriously in terms of the amount of work I would have to put forth. As it was, I knew from the relative start I had work to do.

> Without the interview, I would have been a bit more lax and complacent with this subject matter. I would have been thinking, "But I already know this stuff!"

These quotes illustrate that the interview successfully led PSTs to reflect on their own knowledge and realize that they needed to learn more mathematics. This awareness primed them to engage more diligently in class.

Table 5
Sample Reflection Questions

	Sample reflection questions
1.	I would imagine that the interview with me was kind of an unusual experience for you. Would you agree or not? Explain.
2.	Did this experience evoke any feelings (positive or negative) for you? If so, could you describe them?
3.	Imagine that you have a friend who is going to enroll in this class next semester. If your friend had a choice about being interviewed, would you recommend the interview?
4.	Did you learn something from this experience? If so what?
5.	In the previous question, we asked you what you learned from this experience because we would hope that the person being interviewed would learn from the experience. But the person conducting the interview should also learn. Is there something that you hope we (the interviewers) might learn from the interview we conducted with you? Is there something you hope we did not learn from the interview we conducted with you?
6.	A major goal that I have for this interview is to examine the effect of the interview on the learning process. Do you have any thoughts?
7.	Any other comments/thoughts you would like to share?

Table 6
PSTs' Conceptions at the End of the Course Across Studies 4 and 5 With Interrater Reliability

	Study 4	Study 5
Number of participants	74	23
PSTs with correct conception post (PSTs with correct conceptions pre)	67 (13)	14 (3)
PSTs with incorrect conception post (PSTs with incorrect conceptions pre)	7 (61)	9 (20)
Percentage correct post	91%	61%
Interrater reliability post	93%	96%

Limitations

One limitation of this set of replication studies is that all replications were conducted by the same researcher. More replications are needed to establish whether the effect of an interview at the beginning of the course is similar with different MTEs.

Another limitation is that in most studies, the postconceptions were identified via surveys, resulting in a less in-depth understanding of the categorization of postconceptions in those studies. A replication using postinterviews could yield a more fine-grained understanding of the development of PSTs' conceptions.

"So What?"

When learning of the findings reported in this replication study, many educators (including mathematics educators) seem surprised that so many PSTs do not yet understand multidigit whole numbers when entering their teacher education courses. Understanding numbers is often conflated with "easy" mathematics that everyone is supposed to know and understand. The fact that many PSTs do not yet understand number when they enter their teacher education courses is essential knowledge for both MTEs teaching this population and the PSTs themselves.

The results presented in this Brief Report are important for several reasons. First, they show that the findings of Thanheiser (2009) hold beyond the initial small qualitative study and were replicated across varied populations, time points in education, and geographic locations within the United States. As such, they provide credibility to the original study.

Second, the replication of the findings calls attention to the PSTs' content knowledge at the beginning of the content or methods course. MTEs must understand with what conceptions PSTs enter their classes so that they can create tasks that build on those conceptions. They must also understand which conceptions teachers must develop to be in a position to teach for understanding. As such, the framework and the categorization of PSTs' conceptions of multidigit whole numbers (Thanheiser, 2009) are useful for MTEs concerned with preparing elementary school teachers of mathematics.

Third, the replication studies allowed for the development and validation of a survey to assess the PSTs' conceptions of number. This tool can provide MTEs with quick information about their classes.

Fourth, the replication studies allowed for examining how to address the findings of the original study. The results of the replication show the possibility of leveraging an interview to raise PSTs' awareness of their own knowledge deficits and, thus, motivate them to engage more deeply with the content and improve their learning. By recognizing that they lack mathematics knowledge needed to teach elementary school, PSTs may be motivated to engage with class activities. The data presented show a correlation between the PSTs' recognition of their own limited understanding of mathematics at the beginning of the course and the development of their conceptions. This development may take place because when they recognize the need to learn mathematics, they engage with the activities in the class at a different level.

In addition to the results presented, this Brief Report provides an example of how replication studies can inform the field. In a field in which qualitative studies are common, replications can provide additional information about the phenomena examined. For example, a small-scale qualitative study can be replicated to (a) lend it more credibility, (b) build on the original study to develop additional tools (in this case, a survey), and (c) further examine the phenomenon (in this case, using the interview as a tool to motivate PSTs to engage).

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Author

Eva Thanheiser, Fariborz Maseeh Department of Mathematics and Statistics, College of Liberal Arts and Sciences, Portland State University, Neuberger Hall, Room 323, 724 SW Harrison Street, Portland, OR 97201; evat@pdx.edu

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APPENDIX A

Interview Tasks and Follow-Up Questions

Task 1	**Task 2**
527	389
–135	+475

Ask the PSTs to solve the tasks (printed on separate sheets); ask the PSTs to talk aloud while working, or allow the PSTs to finish all tasks and ask the PSTs to explain how they solved them.

Note. Should students use an algorithm different from one familiar to you, please ask the students to explain their algorithms, and make sure that the students explain and you understand the mathematics of each algorithm. After you have understood the students' solutions, ask them whether they know how to solve the problem a different way. If they do not, then introduce the standard algorithms to them and ask them to make sense of them.

Discussion of 527–135

[After allowing the PSTs to finish their own explanations, depending on the PSTs' answers, choose from the following questions]:

1. "Can you talk a little bit about what you did here [pointing to 527 – 135]?"
2. "What are you doing here [pointing to the regrouping within 527 – 135]? Why does it work?"
3. "What exactly is going on here—in the regrouping part?"
4. Use the PSTs' language: If they say, "I took a 1 and made it a 10," then repeat that and ask what it means. The goal is to understand exactly what they understand.
5. "Did you change the value of the 527?"
6. "What do the *small* numbers mean?"
7. "Why can we simply cross out a number? How and why does this work?"
8. "We can always cross out the number to the left, make it one smaller, and put a 1 by the number next to it on the right. Can you explain why that [procedure] works?"

Discussion of 389 + 475

1. "Solve 389 + 475 and discuss."
2. "Can you explain this [the regrouping] in more detail?" Point to the ones and ask what they did there.
3. "Talk about these [the two 1s—DO NOT SAY "Ones." Instead, point]."
    i. "What does this 1 [regrouped to tens] represent?" Do NOT say "one" but point to it!

ii. "What does this 1 [regrouped to hundreds] represent?" Do NOT say "one" but point to it!
iii. "Compare the two [1s]. Are they the same or are they different?"
iv. For each regrouped 1, ask, "One what?" Pose this question only as a last resort!

APPENDIX B

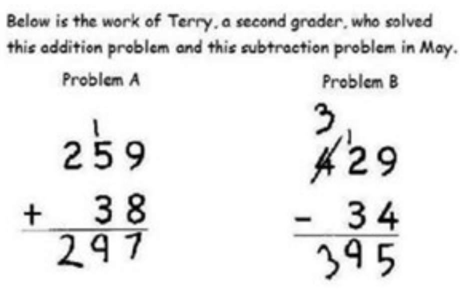
Survey Questions

1. Please consider the regrouped 1s in the problem below:

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 389 \\ \underline{+475} \\ 864 \end{array}$$

   a. What does the 1 above the 8 represent? Please be as specific as you can.
   b. What does this 1 above the 3 represent? Please be as specific as you can.
   c. Compare the two 1s. Are they the same or are they different? Please be as specific as you can.

2. Please answer the questions below:



   a. Does the 1 in each of these problems represent the same amount? Please explain your answer and be as specific as you can.
   b. Explain why in addition (as in Problem A) the 1 is added to the 5, but in subtraction (as in Problem B), 10 is added to the 2.