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ON THE CONCEPT OF DOCUMENTATIONAL ORCHESTRATION

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Taking as a basis the concepts of instrumental orchestration (Trouche, 2005a) and documentational genesis (Gueudet & Trouche, 2009), in this paper I try to articulate the concept of documentational orchestration. I argue that especially in mathematics teacher education is worthwhile to develop this concept because it addresses the problem of designing activities for teachers, offering a particular way of observing some of the effects or consequences of a particular design, an also a way of guiding the refinement and improvement of such design. This guide is based on the location and observation of the instrumentalization and instrumentation processes that may take place during the application of a particular design. The utilization of the concept is illustrated through the application of an orchestration in an Internet-based teacher education program.

INTRODUCTION

The concept of *documentational genesis* (Gueudet & Trouche, 2009) is a new theoretical concept that seems to be a useful analytical tool for studying the development of mathematics teachers. The concept of *documentational genesis* can be considered as an analogy of the concept of *instrumental genesis* (Rabardel, 1995; Trouche, 2005b) into the field of mathematics teacher education. In this new approach the focus is on the activities that the teacher develops outside the classroom, but that influence his work in the classroom. In particular the focus is on the *documentation work* of the teacher, i.e. the interaction of a teacher with a set of elements that shape and define his work in the classroom; for example, to draw examples and exercises out from a textbook for his mathematics lesson plans, to look up his own notes of previous courses, to analyze the mathematical productions of his students, to listen to suggestions or criticisms of his fellow teachers and his students, to study a curriculum reform to be implemented in his school, etc. This set of elements with which the teacher interacts in order to carry out his documentation work is called *resources*.

There are situations where the interaction between mathematics teachers and resources is not spontaneous. In these contexts there is a need for organizing and arranging the set of resources that the teachers interact with, in order to develop specific aspects of their professional knowledge. Here I am referring to the in-service mathematics teacher education programs. Inspired by the concept of instrumental orchestration (Trouche, 2004; Trouche, 2005a), the aim of this paper is to illustrate and to argue that at least in the field of mathematics teacher education, it make sense and is relevant to use and to develop the concept of documentational orchestration. I will highlight the necessity of studying in a joint way this concept together with the concept of documentational genesis. Particularly it is shown how the instrumentalization and instrumentation processes that constitute a documentational genesis, can be taken as a basis to guide the refinement and redesign of an orchestration. The empirical evidence supporting the arguments are taken from an *online* educational program for mathematics teachers, that is to say, the structure and operation of the program is based on the use of the Internet and its associated tools.

SOME THEORETICAL CONCEPTS

Documentational genesis

The work of Gueudet & Trouche (2009) suggests a way to "trace" the professional development of mathematics teachers. To achieve this, the authors suggest to focus our attention on the sort activities that teachers develop outside the classroom. The focus should be particularly centered on teacher's *documentational work*; that is to say, the interaction of a teacher with a number of elements that allow him to shape and to define his work in the classroom. The set of elements that a teacher interacts with during his documentational work is called *resources*. Resources can be constituted by elements of a different nature such as textbooks, web pages, personal notes, a particular piece of software, a talk with a fellow teacher, student responses to a mathematical task, and so on.

In this new approach it is claimed that when a teacher interacts with a set of resources a *documentational genesis* (DG) may occur. The concept of DG can be considered as an analogy of the concept of *instrumental genesis* (Rabardel, 1995; Trouche, 2005b) applied in the field of mathematics teacher education. Like the instrumental genesis, a DG is also a two-way process in which a teacher appropriates and/or modifies the set of resources that he is interacting with (this part of the process is called *instrumentalization*), but also the resources shape and influence the activity of the teacher (this part of the process is called *instrumentation*). Thus, through a DG the teacher can develop a *document* from the set of resources he interacted with.

An example of a *document* is presented in Gueudet & Trouche (2009, p. 205). In this example a teacher faces a particular *class of professional situations* (Rabardel & Bourmaud, 2003), namely, "propose homework on the addition of positive and negative numbers". After looking at several resources such as textbooks and a list of exercises that she has used in previous courses, the teacher builds a new list of tasks that she uses in her classroom. The list of tasks could be modified by the teacher after seeing how it works in her classroom, and might even be reused in a new group of students or during the next school year. After looking at this example one could interpret that the document created by the teacher is the list of mathematical tasks that she produced, however a document is not necessarily a physical entity.

A document is a mental scheme (also called *scheme of utilization*) that is associated with a specific set of resources (in the previous example, the textbooks and the exercises list she consulted) that guides and defines teacher's actions for a given class of situations (in this case, to propose homework about the addition of positive and negative numbers), through different contexts (the group where she applied the list of tasks and the future possible groups and courses where she could reuse the list of tasks). In the example mentioned above, the list of tasks is just a visible part of the constituted document. There are other non-visible elements that guided and determined the selection and design of the tasks that the teacher listed. Those elements are beliefs and implicit values that drive and lead teacher's action; Gueudet & Trouche (2009) mention an example of these non-visible elements: the idea that "the additions proposed must include the cases of mixed positive and negative numbers, and of only negative numbers".

Thus, a document is associated with a specific set of resources and is composed of a visible and tangible part called *usages*, and an implicit and non-visible part called *operational invariants* (Vergnaud, 1998). A document can then be expressed by the following formula:

Document = Resources + Usages + Operational Invariants

Due to its implicit nature, the operational invariants can not be observed directly. They can be inferred from the prolonged observation of teacher's action. The identification of regularities in the teacher practice across different contexts can facilitate the inference and interpretation of the operational invariants that guide the practice.

Instrumental orchestration

In the instrumental approach it is claimed that the schemes of utilization have a social dimension. It is said that the schemes of utilization are developed and shared in communities and may be even the result of explicit training processes. Thus, it is necessary that these "explicit training processes" could be carefully designed to encourage the establishment or modification of schemes of utilization. It is in this point where the concept of *instrumental orchestration* appears. It refers to the organization of the artifactual environment, which an institution designs and puts in place, with the main objective of assisting the instrumental genesis of individuals (Trouche, 2005a, p. 210).

A instrumental orchestration is defined by two elements (Trouche, 2005a, p. 211):

- A set of *configurations* (i.e. specific arrangements of the artifactual environment, one for each stage of the mathematical situation);
- A set of *explotation modes* for each configuration.

Documentational orchestration

Let's move now to the mathematics teacher education context. This is a context where teacher educators have a set of goals or educational purposes, but they also have a set of resources to try to achieve those goals.

In this context is important to explicitly discuss what the pursued objectives are, and whether the different arrangements or accommodations of the resources are appropriate to achieve those goals. It is here where I find relevant to introduce and to use the concept of *documentational orchestration* (DO). A DO can be defined as an arrangement or accommodation of resources that a teacher educator (or a group of teacher educators) performs with the intention of facilitating and encouraging the documentational work of mathematics teachers, aiming at contributing to the development of their professional knowledge. In principle, the structure of a DO should include the two elements that define an instrumental orchestration, i.e., configurations and exploitation modes. These two elements must be defined in terms of the possibilities and limitations of the educational setting where the orchestration will be applied; it has to be also taken into account the type of knowledge we want to produce.

I think the concept of DO can help us to discuss in an explicit and organized way the relations between pursued aims and the arrangement of resources. In addition, an DO can be refined or redesigned through the identification of the instrumentalization and instrumentation processes that might occur during the implementation of the orchestration.

I will illustrate the application of the concept with an example that has been designed for and implemented in an internet-based in-service teacher education course. In order to design a documentational orchestration it is necessary to specify the environment in which the orchestration will be organized as well as the aim of the orchestration. In the next section I refer to those two elements.

AN EXAMPLE OF A DOCUMENTATIONAL ORCHESTRATION

About the setting where the orchestration was applied

The documentational orchestration was applied in an internet-based educational program for in-service mathematics teachers. This is a program[3] based in the *Instituto Politécnico Nacional* of Mexico. The program offers a master degree in mathematics education (two years). The technological nature of this program has helped to eliminate temporal and geographical barriers, allowing teachers from all over Latin American to have access to this educational program.

To implement the courses that constitute this educational program, is used *Moodle* (http://moodle.org). This is a free and open source platform that allows you to arrange courses by storing and sharing different types of files (such as text, audio and video files), but also permits to organize asynchronous discussions among the participants of a course. An asynchronous communication is the one that is carried out mainly by means of an exchange of written messages between two or more people, but the answers or reactions that the participants get are not immediate, for example, you can raise a question or an observation and get the feedback or reactions to it several minutes or hours after. The asynchronous discussions usually last several days, allowing the participants to have more time to formulate their opinions and to reflect on comments and opinions expressed by the other participants. It is even possible to consult external sources in order to enrich and clarify a discussion in an asynchronous communication. The email messages and the discussion forums are some examples of asynchronous communication.

The aim

The data used in this paper were taken from a course on the use of technology for the teaching of the mathematics. The course lasted for four weeks and it was carried out during November and December 2008. The course was attended by four mathematics teacher educators and fourteen mathematics teachers coming from Mexico and Argentina. The main objective of the orchestration was to make teachers aware of the possible modifications or changes that the tasks and the techniques can experience when technology is introduced in the mathematics classroom as a study tool. We were particularly interested in teachers noticing that a) new techniques may emerge, i.e., techniques that rely on the use of technology, and b) that some tasks and techniques could lose its meaning and become obsolete. Along the course and also in this writing the terms *tasks* and *techniques* are used in the sense of Chevallard (1999).

Didactical configuration

Here I refer to the specific arrangements of the resources with which the teachers interacted during the course. The configuration was aimed at promoting teachers' awareness about the possible effects that produces the use of technology on mathematical tasks and techniques. The configuration is divided into five stages. Figure 2 shows a graphical representation of such configuration.

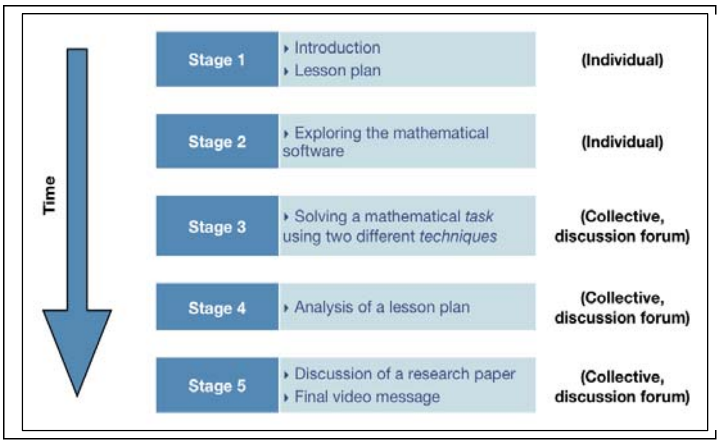

Figure 2: Graphical representation of the didactical configuration.

The concepts of task and technique are the guiding structure of the configuration. The configuration rests on locating those two elements in a math lesson plan designed for a paper and pencil environment (stage 1), and reflect and discuss about their pertinence in a mathematics software environment (stage 4). The discussion about the pertinence should take place after teachers themselves experience some instrumented techniques (stages 2 and 3). The last layer of the configuration is an 'institutionalization' stage.

In **stage 1** an introduction to the course was done. Teachers were notified that the structure of the course was based on the concept of praxeology (see Chevallard, 1999), and by means of an example the components of a praxeology were illustrated. The example used describes a teacher who introduces in her class the topic "quadratic functions". One of the *tasks* that the teacher presents to her students is to "find the roots of $f(x) = ax^2 + bx + c$" To solve this *task* the teacher presents a particular *technique* to her students, consisting in applying the quadratic formula: $x_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The teacher explains how to interpret the terms a, b and c of the previous expression. She also shows through examples, that it is always possible to determine the roots of the quadratic function by applying the formula. This speech that the teacher uses to introduce and to illustrate the use of the *technique*, is called *technology*. Thus, some students can successfully apply the technique, but probably they do not understand why the formula always works. The mathematical theory that explains and supports the operation of the technique, and that probably at this stage of their education exceeds the mathematical understanding of students, is what is called *theory*.

The first activity of the course for the teachers was to locate a mathematical topic that they have already taught or that they liked to teach. Afterwards they should identify the type of tasks and techniques that they usually present to their students when they introduce the mathematical topic. This lesson plan was requested at the beginning of the course, to avoid any influence of the contents of the course on their lesson plans.

The course was intended to use a piece of mathematical software. Teachers were provided with a copy of this software. In **stage 2** which lasted three days, teachers were solving specific mathematical tasks using the software. The secondary objective of the activity was to help teachers become familiar with the software, but particularly with the CAS and graphical capabilities of the tool.

The **stage 3** was inspired in the work of Mounier and Aldon (1996) presented in Lagrange (2005). Teachers were organized in teams of four or five members, and then each team was asked to be split into two sub-teams. Both sub-teams should find a general factorization for the expression $x^n - 1$, but one team should only use paper and pencil, while the other one should only utilize the command *Factor* from the mathematical software. At the end of the stage both sub-teams should share their results and to discuss which technique was better to solve that sort of mathematical task. The conclusions of the each team conclusions were presented in a written report. Both the solution of the mathematical activity and the discussion of the results were collectively carried out in an asynchronous discussion that lasted seven days. The secondary objective of this stage was to allow teachers experience different techniques and discuss their differences, advantages and disadvantages. Teachers were expected to highlight the pragmatic value of techniques (Lagrange, 2005), for example, the speed and efficiency with which the software makes the factorization. But it was also expected that teachers (and particularly those who worked with the command *Factor*) recognized some kind of epistemic value in the instrumented techniques.

Stage 4 and stage 1 are linked. During the fourth stage one of the lesson plans that teachers prepared in stage 1 was selected. A lesson plan was selected concerning the solution of systems of linear equations in two variables. The proposed tasks were to find the solution of different systems of linear equations, and the offered techniques were the addition, substitution and graphical solving methods (using only paper and pencil). I was authorized by the teacher who wrote this lesson plan to use it as part of the stage 4.

In this stage teachers were divided into teams and each team was assigned to an asynchronous discussion forum. The selected lesson plan was presented to them in the discussion forum, together with the following hypothetical situation: "There is a mathematics class where students are allowed and know how to use the mathematical software used in stages 2 and 3. If you apply the selected lesson plan in such mathematics classroom and students start to use the software for solving the mathematical tasks, then what kind of effect does technology have on the tasks and techniques included in this lesson plan?" The secondary objective was to highlight some of the effects that technology may have on mathematical tasks and techniques. For example, some of the proposed techniques will become obsolete, because there will be other (instrumented) techniques that would do the work in a more quickly and efficient way. If they perceived this, then it was also expected that teachers felt the need for redesigning the lesson plan in order to implement it in a setting supported by the use of technology.

The **fifth** stage was a moment of institutionalization of the contents of the course. The teachers and teacher educators who participated in the course discussed in an asynchronous forum the content of the research paper Lagrange (2005). It was initially planned to focus the discussion of the paper on the modifications on tasks and techniques that the author of the paper reports. Additionally, a video message was published. In this message the secondary objectives of each one of the activities that integrated the course were explicitly mentioned. This video served as a mean to bring into an explicit level all the ideas that were implicitly involved in the prior stages.

Exploitation modes

The exploitation modes refer to the possible adjustments of the variables of the established configuration. These adjustments should be guided by the intentions of the designer or teacher educator, and also by the purpose of the orchestration. For example, in the configuration previously presented different exploitation modes are possible:

- The configuration privileges the study of CAS and graphical techniques. Adjustments on the mathematical activities of stages 1 and 2 would allow us to shift the focus to techniques and tasks related to dynamic geometry or spreadsheet software, for example.

- In the stage 4, it is not compulsory to have a lesson plan designed by a teacher. The lesson plan can be planned in advance to suit the intentions of the designer. In this way we could cover the analysis of a variety of tasks and techniques suitable for different educational levels.

- The interaction of the teachers with a teacher educator is another variable of the configuration. The collective stages 3, 4 and 5 allow the participation of teacher educators in the discussion forums. A teacher educator can help to promote, to moderate and to guide the discussion; nevertheless sometimes is convenient to establish a discussion where teachers engage in a discussion that is free of the authority of the teacher educator.

DISCUSSION

The concept of documentational orchestration enables us to bring into an explicit level the sort of knowledge (or document) that we want to produce, and the way in which the resources are organized in order to reach that aim. One could argue that this explicit and orderly way to analyze the arrangement of resources and its relation to the aim of the design could be accomplished without using the concept of documentational orchestration, but this is not a concept that should be considered in isolation. The theoretical strength of the concept lies in its connection with the concept of documentational genesis. We can not use the concept of documentational orchestration without making reference to the documentational genesis concept.

A documentational orchestration is regulated and evolves through the feedback that is obtained during its application. This 'feedback' is represented by the instrumentation and instrumentalization processes that are manifested in different stages of the orchestration (see figure 1). Let me introduce two examples of the manifestation of those processes:

Example 1, an instrumentalization process. The following is an excerpt from an asynchronous discussion forum from stage 3. In this forum a Mexican and an Argentinean teachers are trying to find a general characterization for the algebraic expression $x^n - 1$. The Argentinean teacher mentioned that she has been implementing

Ruffini's rule during her explorations, and then her Mexican colleague asked her in what book he could find Ruffini's rule. This is the answer to that question (the real names of the teachers have been replaced to protect their identity):

Topic: Re: Team 2. "Paper and pencil technique"
From: Norma
Date: Wednesday, 26th of November 2008, 00:09

Nice to meet you Homero, how are you?

You might already know the Ruffini's rule (as we call it here [in Argentina]) but with another name, it is a shortened way of solving divisions having the form P=(x) / (x+-b) […] To be consistent with this course, I will not recommend you any books, I give you the link to a youtube video.

A picture is worth a 1000 words, don't you think? ☺

http://es.youtube.com/watch?v=RViiUlWty8M

Best wishes, Norma

This is a clear example of an instrumentalization process in which the teacher introduces an innovation in the resources. The teacher uses a link to a YouTube video as a tool to communicate a mathematical idea to one of her colleagues. Even though ourselves (the teachers educators) had previously used this website to post messages on video, this was the first time we saw a teacher using this site as a mean to communicate mathematical ideas.

Example 2, an instrumentation process. After analyzing the asynchronous discussions that teachers produced on the stages 3 and 4 of the orchestration, it became clear that only a few of them highlighted the pragmatic value of the instrumented techniques. In other words, teachers conceived the software as a tool that facilitates the implementation and verification of algorithms, but not as a tool that can serve as a mean to produce mathematical knowledge. Such positions can be illustrated by some of the comments made by the teachers. For example, during the third stage, when the sub-teams had to be defined, one teacher commented:

Topic: Re: General discussion space
From: Sandra
Date: Monday, 24th of November 2008, 15:44

Hello colleagues. We have to define the sub-groups to solve the activity 3.

If you ask for my opinion, I would like to work with paper and pencil. Who else would like to join me? I will wait for your answers

Best wishes to all, Sandra

Then one of the teachers reacted to Norma's comment:

Topic: Re: General discussion space
From: Federico

Date: Monday, 24th of November 2008, 19:35

Hello Sandra.

Hi Sandra, even though I support the use of calculators, I am convinced that the proper use of calculators requires prior understanding about how the things are done. I would like to team up with you, if you agree we could do it. I am open and willing to see other colleagues' points of view.

Best wishes, Federico

My interpretation of the phrase "I am convinced that the proper use of calculators requires prior understanding about how the things are done" is that this teacher perceives technology (in this case, calculators) as an element whose use in the classroom should be subsequent to the work with pencil and paper. This teacher does not perceive the instrumented techniques as a mean to produce knowledge. This idea or position is interpreted here as a component of the operational invariants that this teacher associate with the use of technology to teach mathematics.

For the teacher educators who were observing the teachers' discussions, it was clear that after the teachers had passed through the initial stages of the orchestration, most of them only highlighted the value of pragmatic techniques implemented without mentioning any epistemic value. This issue was explicitly addressed during a meeting that teacher educators held three days after the fourth stage of the orchestration started. This meeting was supported by the use of the software *Skype*. In this meeting we decided that during the fifth stage in the orchestration, where we should discuss with the teachers the work of Lagrange (2005), we will focus the discussion of the concepts pragmatic and epistemic values. In some cases the discussion was very productive. For example, the teacher who was quoted above, mentioned:

Topic: Re: What technology in the mathematics classroom?
From: Federico
Date: Saturday, 13th of December 2008, 04:16

Hello colleagues

Before reading the article of Lagrange I just gave one application, using the terminology of the article, pragmatic. I felt that without a prior knowledge, the use of tools as CAS and/or calculators do not help to generate learning, I mean, I was in favor of using these tools, but apparently I was just giving them a pragmatic value. On integral calculus I promoted the use of tools for the calculations and at the most in the derivative calculations. On differential equations I promoted its use to carry out integrals and so on. So I am very surprised that the article highlights the aspect of the epistemic application. In a sense he was right, because the epistemic application obviously requires a planning and construction of new activities that do not arise naturally from the teaching based on pencil and paper. I would like to finish this comment, leaving the thought and concern of how should be a methodology for applying the epistemic value.

Best wishes, Federico

I interpret this comment as evidence that there has been a change in the operational invariants that the teacher associated with the use of technology, a change that seems to have been motivated by some elements of the set of resources that the teacher interacted with, particularly the concepts of value epistemic and pragmatic value of a technique presented in the article by Lagrange (2005).

Final comments

As I mentioned before a DO is regulated and evolves through the information that the instrumentalization and instrumentation processes provide. However, the type of information that the designer gets about his orchestration, is different for each of these processes. The instrumentalization processes help us to identify the resources that are appropriated, modified or introduced by the teachers. This allows us to see the consequences of these changes and to take them into consideration for improving future orchestrations. In the first example a teacher uses a YouTube video as a mean to communicate mathematical ideas. This particular way of using this type of videos was new even for us the teacher educators. This has been a trigger that has made us reflect on the different uses and functions that could have such resources in future orchestrations.

The information provided by the instrumentation processes is less general. The presence or absence of these processes reveals whether or not the primary and secondary objectives of the orchestration are being achieved. This information allows us to make adjustments and specific modifications to the stages and exploitation modes of the DO with the intention of improving it. The example 2 illustrates this process.

An important idea that has remained implicit in the paper is the iterative or cyclical nature of a DO. Here I am claiming that as the documentacional genesis, an DO can be seen as a process in which an orchestration is applied and its application produces (or does not produces) certain instrumentalization and instrumentation processes, then, taking into account these processes, the orchestration can be redesigned or transformed into a new orchestration. This cyclical nature of the design of tasks in teacher education has been mentioned by other teacher educators (see for example Yackel, Underwood & Elias, 2007; Liljedahl, Chernoff & Zazkis, 2007). One of the main contributions offered by the concept of DO to this discussion is the proposal to focus our attention to the processes may arise during the implementation of an orchestration, and use them as a source of information that can serve as guide for adjusting the original design.

NOTES

1. This work was supported by the Programme Alβan, the European Union Programme of High Level Scholarships for Latin America, scholarship No. E06D101377MX.

2. An extended and different version of this paper has been submitted for evaluation to the journal *Recherches en Didactique des Mathématiques*. This new version of the paper, which has been written in spanish, includes fresh empirical data regarding the instrumentation and instrumentalization processes.

3. More information about this educational program can be found in www.matedu.cicata.ipn.mx (in Spanish).

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While the didactics of mathematics and science has always been about understanding and improving the teaching of these subjects, cognitivist paradigms for a long time made the descriptive or “pure observational” part of this endeavour somewhat dominant.

Increasingly, however, didactical research is based on designed interventions. With developments such as didactical engineering or learning demand studies, didactics is increasingly becoming a design science. The papers in this volume explore these new developments in a variety of areas.

All papers were written by participants in the PhD-course “Didactics as design science”, the first in a series of four international PhD-courses offered by FUKU (The Graduate Programme on Education of the University of Copenhagen) and the Department of Science Education.

The papers were peer reviewed after the course and reflect the high quality of the participants’ work.

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